# 4.2 - Translations of Sine and Cosine

## Warmup

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Find the exact value.

1) $\dfrac{\sin 30^\circ + \cos 60^\circ}{2}$

$\dfrac{1}{2}$

2) $\dfrac{4\sin 300^\circ + 2\cos 30^\circ}{3}$

$-\dfrac{\sqrt{3}}{3}$

3) $4(\sin 30^\circ)(\cos 60^\circ)$

$1$

4) $\sin 30^\circ + \sin 60^\circ$

$\dfrac{1+\sqrt{3}}{2}$

5) $(\sin 60^\circ)^2 + (\cos 60^\circ)^2$

$1$

6) $8(\sin 120^\circ)(\cos 120^\circ)$

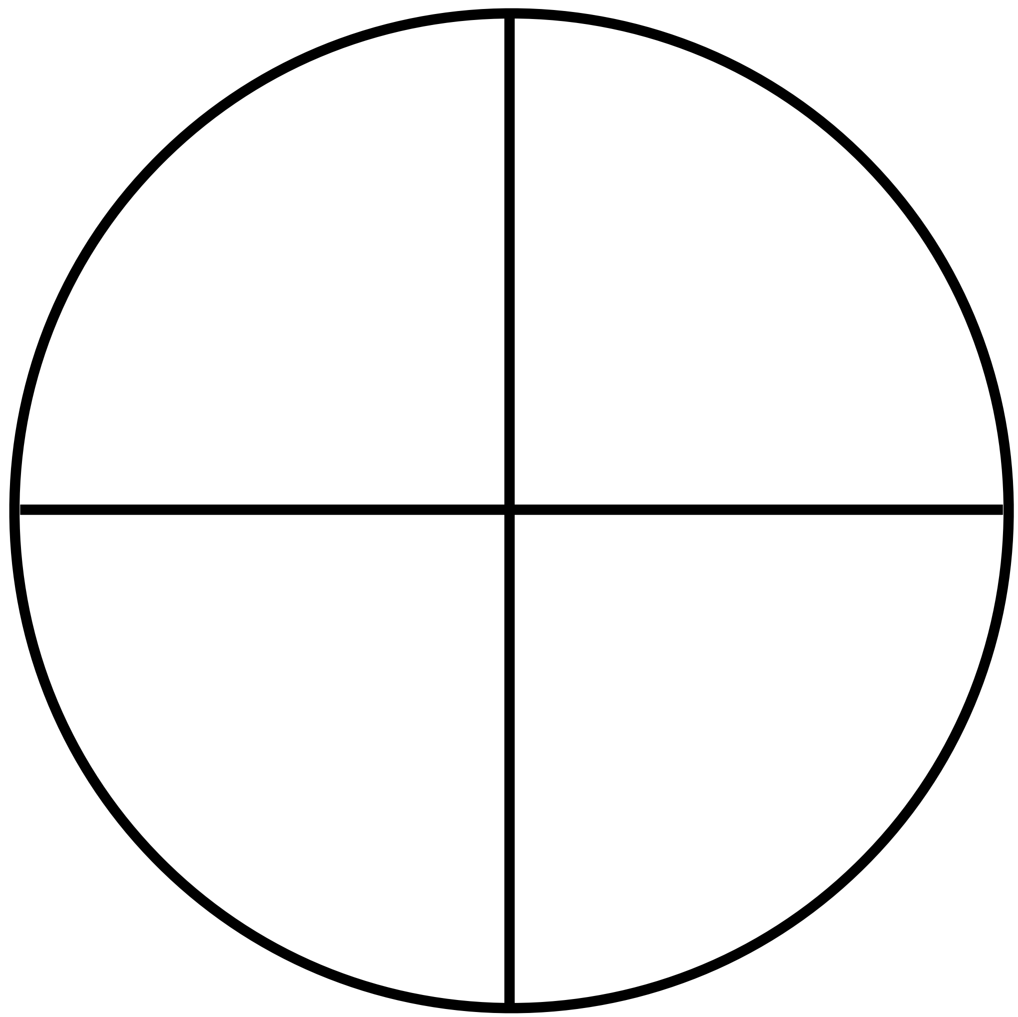
$-2\sqrt{3}$

# 4.2 - Translations of Sine and Cosine



## The Unit Circle

Trig values

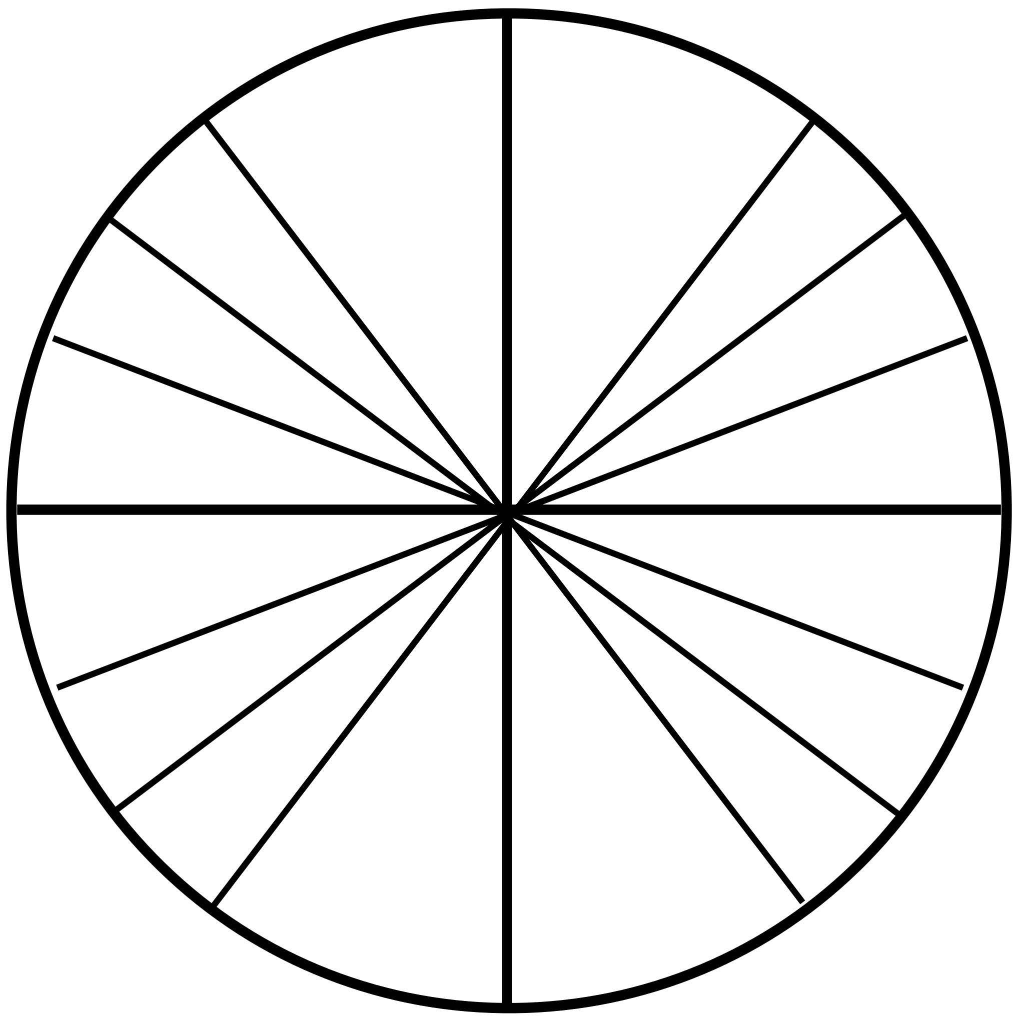


# 4.2 - Translations of Sine and Cosine



## The Unit Circle

Trig values



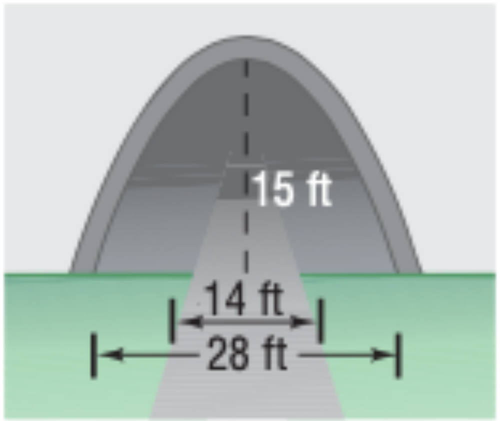
# Trig Word Problems

# 4.2 - Translations of Sine and Cosine



## Sample Problem

**92. Bridge Clearance** A one-lane highway runs through a tunnel in the shape of one-half a sine curve cycle. The opening is 28 feet wide at road level and is 15 feet tall at its highest point.




$$h = ?\sin(?x) + ?$$

$$h = 15\sin(\frac{2\pi}{56}x) + 0$$

(a) Find an equation for the sine curve that fits the opening. Place the origin at the left end of the sine curve.

(b) If the road is 14 feet wide with 7-foot shoulders on each side, what is the height of the tunnel at the edge of the road?

$$h = 15\sin(\frac{2\pi}{56} \cdot 7) + 0$$

$$h = 10.6\ ft$$

# 4.2 - Translations of Sine and Cosine



## Practice

**Motion of a Buoy** A signal buoy in the Chesapeake Bay bobs up and down with the height $h$ of its transmitter (in feet) above sea level modeled by $h = a \sin bt + 5$. During a small squall its height varies from 1 ft to 9 ft and there are 3.5 sec from one 9-ft height to the next. What are the values of the constants $a$ and $b$?

$$h = ?\sin(?x) + 5$$

$$h = 4\sin\left(\frac{2\pi}{3.5}t\right) + 5$$

# 4.2 - Translations of Sine and Cosine



## Practice

**Motion of a Buoy** A signal buoy in the Chesapeake Bay bobs up and down with the height $h$ of its transmitter (in feet) above sea level modeled by $h = a \sin bt + 5$. During a small squall its height varies from 1 ft to 9 ft and there are 3.5 sec from one 9-ft height to the next. What are the values of the constants $a$ and $b$?

a) What is the height after 2.5 seconds?

$h = 1.1\ ft$

b) How long before the height is 8 feet the first two times?

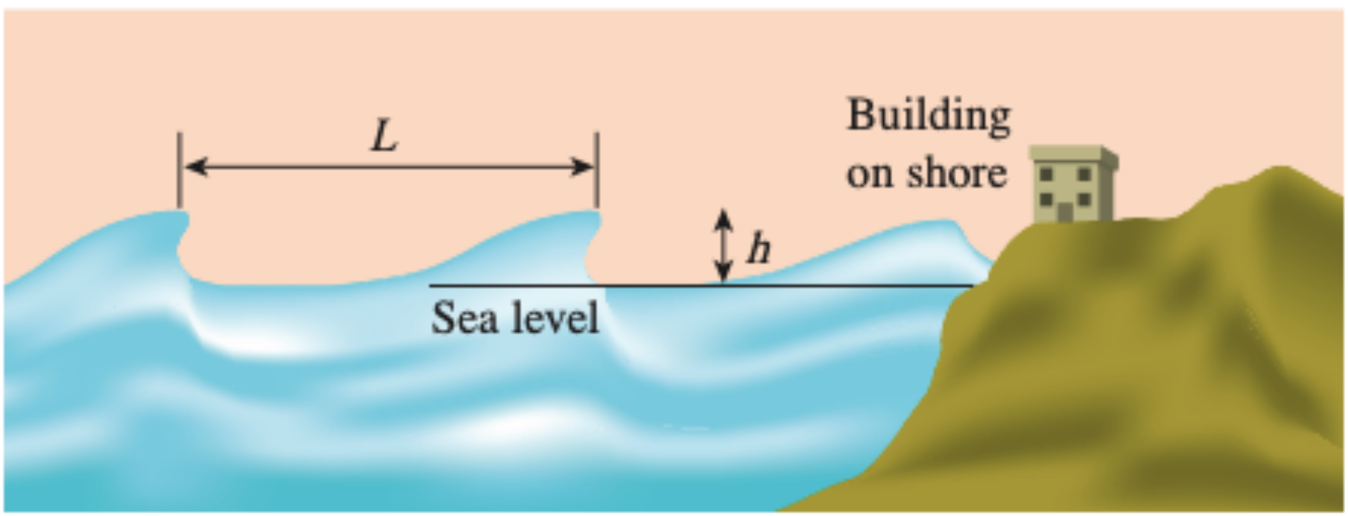
$t = 0.47\ secs,\ 1.28\ secs$

# 4.2 - Translations of Sine and Cosine



## Practice

**Tsunami Wave** An earthquake occurred at 9:40 A.M. on Nov. 1, 1755, at Lisbon, Portugal, and started a *tsunami* (often called a tidal wave) in the ocean. It produced waves that reached a height of 60 ft. If the period of the waves was 30 min or 1800 sec, The period length is 100 ft. Find the equation of the height relative to the time (sec).




$$h = 30\sin\left(\frac{2\pi}{1800}t\right) + 30$$

# 4.2 - Translations of Sine and Cosine

## Practice

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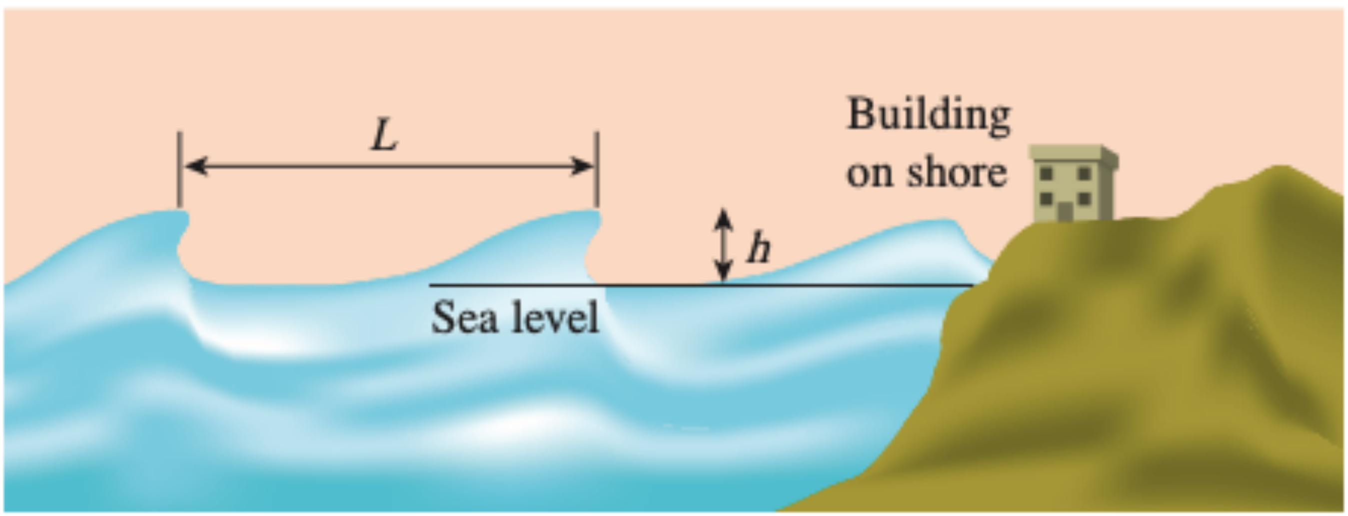
**Tsunami Wave** An earthquake occurred at 9:40 A.M. on Nov. 1, 1755, at Lisbon, Portugal, and started a *tsunami* (often called a tidal wave) in the ocean. It produced waves that reached a height of 60 ft. If the period of the waves was 30 min or 1800 sec, The period length is 100 ft.

How long (hours) before it travels one mile (5280 ft)?




$$h = 30\sin\left(\frac{2\pi}{1800}t\right) + 30$$

$$5280\ ft \cdot \frac{0.5 hours}{100\ ft} = 26.4 hrs$$

# 4.2 - Translations of Sine and Cosine



## Practice

**Ferris Wheel** A Ferris wheel 50 ft in diameter makes one revolution every 40 sec. If the center of the wheel is 30 ft above the ground find the equation of the height relative to the time.

$$h = -25\cos\left(\frac{2\pi}{40}t\right) + 30$$

$$h = 25\sin\left(\frac{2\pi}{40}(t-10)\right) + 30$$

# 4.2 - Translations of Sine and Cosine

## Practice



**Ferris Wheel** A Ferris wheel 50 ft in diameter makes one revolution every 40 sec. If the center of the wheel is 30 ft above the ground, how long after reaching the low point is a rider 50 ft above the ground?

$$h = -25\cos\left(\frac{2\pi}{40}t\right) + 30$$

$$h = 25\sin\left(\frac{2\pi}{40}(t-10)\right) + 30$$

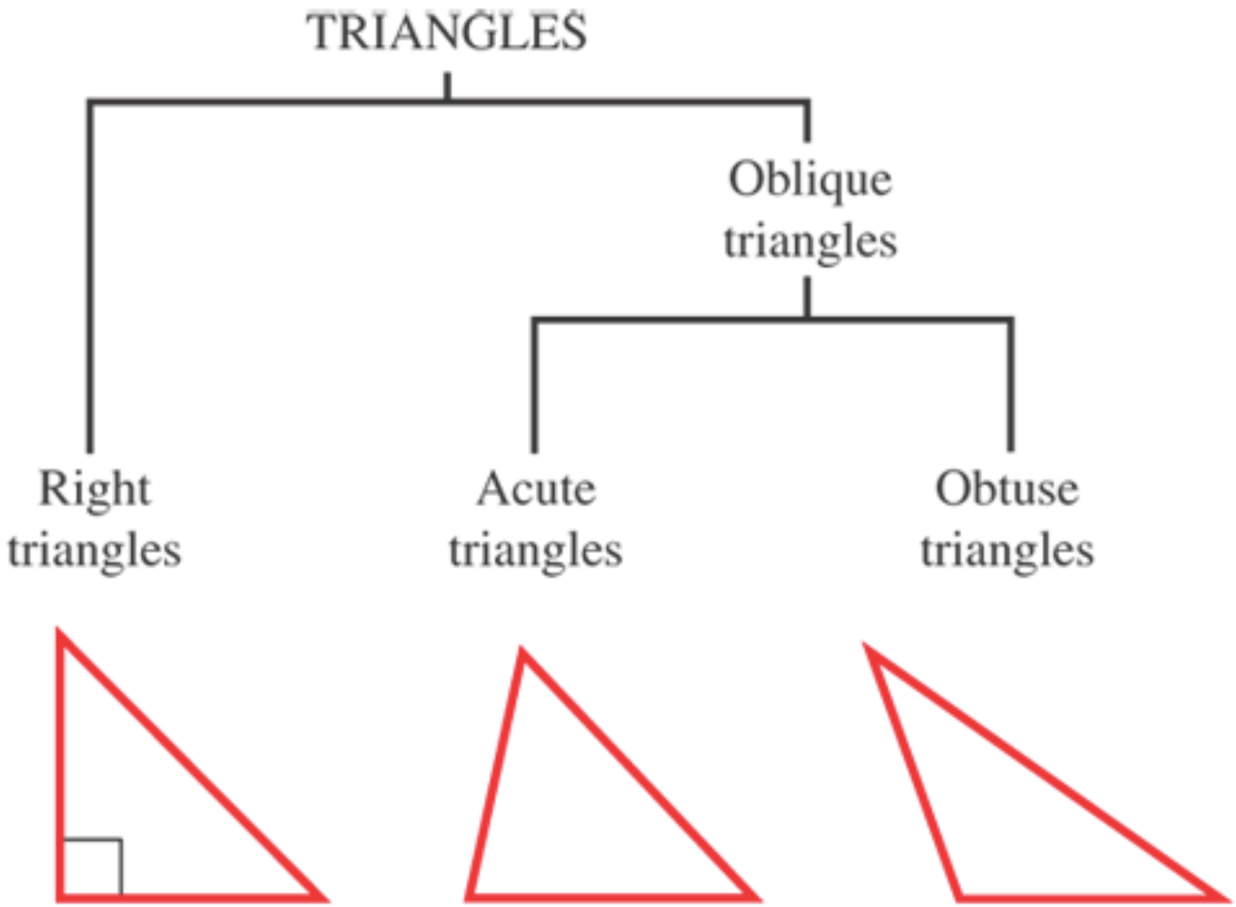
$$t = 15.9\ sec,\ 24.1\ sec$$

# Law of Sines

# 7.1 - Law of Sines



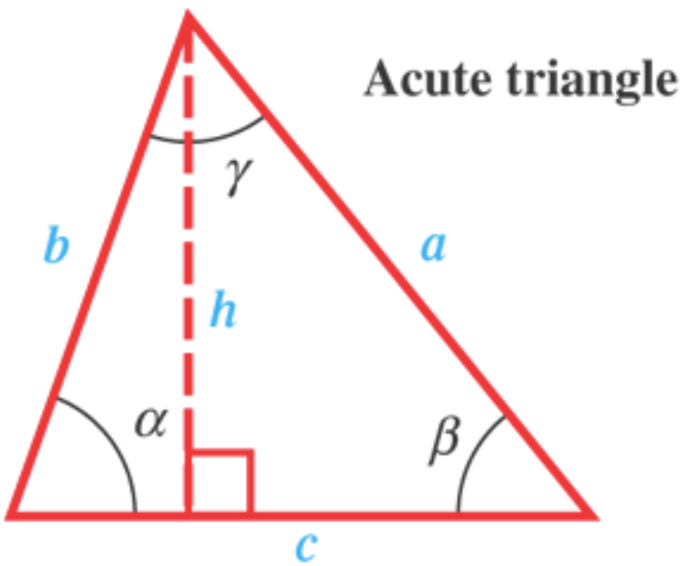
**Oblique Triangle**: Any triangle that does not have a right angle.

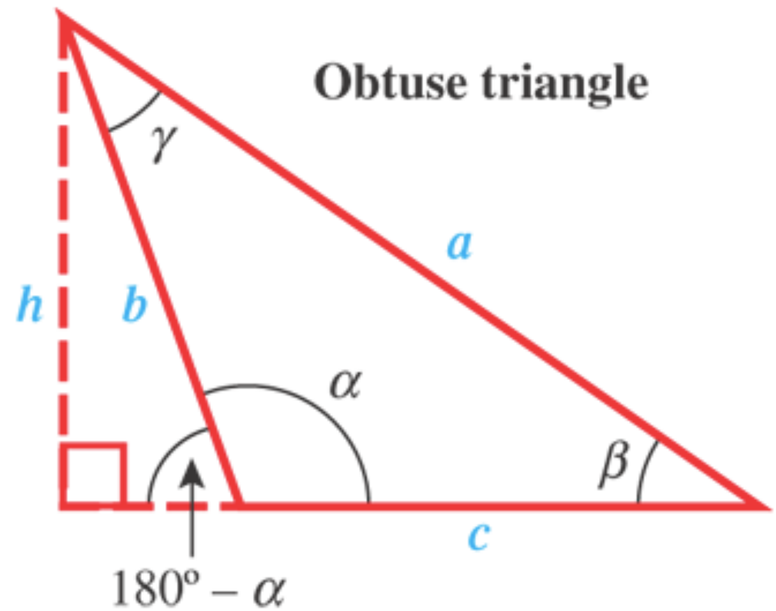


# 7.1 - Law of Sines



**Sine Ratios**:




$$\sin\alpha = \frac{h}{b} \qquad \sin\beta = \frac{h}{a}$$

$$\sin(180^\circ - \alpha) = \sin\alpha$$

$$\sin\alpha = \frac{h}{b} \qquad \sin\beta = \frac{h}{a}$$

**SAME!!!**

# 7.1 - Law of Sines

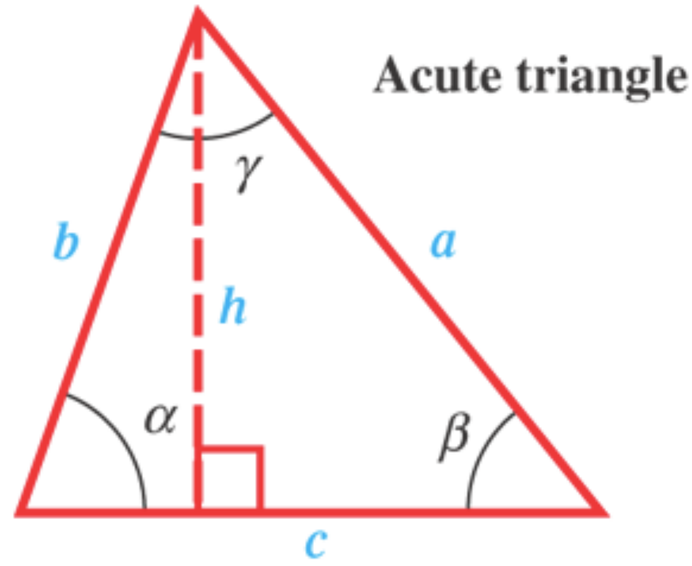


$$\sin\alpha = \frac{h}{b} \qquad \sin\beta = \frac{h}{a}$$

$$h = b\sin\alpha \qquad h = a\sin\beta$$

$$b\sin\alpha = a\sin\beta$$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$




$$\boxed{\frac{\sin\alpha}{\mathbf{a}} = \frac{\sin\beta}{\mathbf{b}} = \frac{\sin\gamma}{\mathbf{c}}}$$

**Law of Sines**

# 7.1 - Law of Sines



Law of Cosines is good for solving:

- SAS
- SSS

$$\mathbf{c}^2 = \mathbf{a}^2 + \mathbf{b}^2 - 2\mathbf{ab}\cos\mathbf{C}$$

Law of Sines is good for solving:

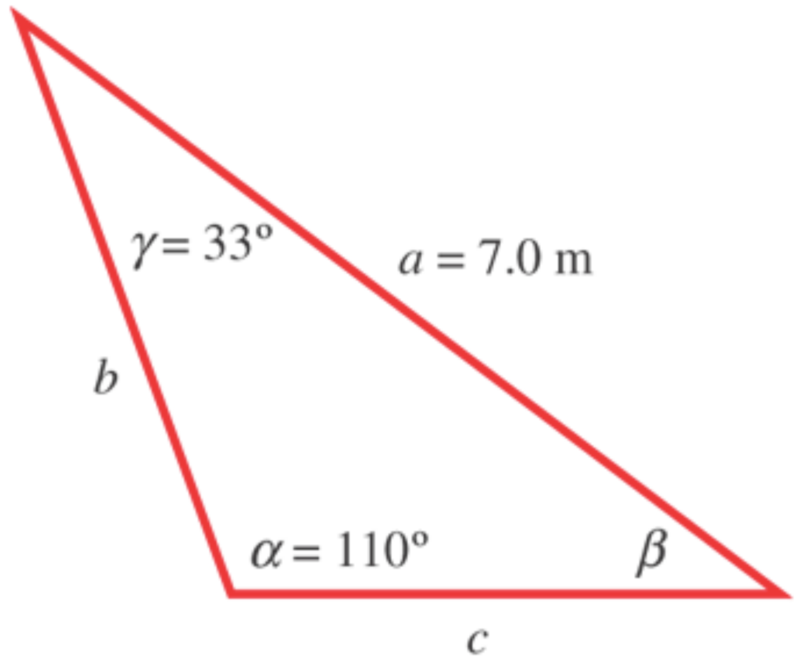
- AAS
- ASA
- SSA

$$\frac{\sin\alpha}{\mathbf{a}} = \frac{\sin\beta}{\mathbf{b}} = \frac{\sin\gamma}{\mathbf{c}}$$

# 7.1 - Law of Sines



Solve the triangle (AAS)




$\beta = 37°$

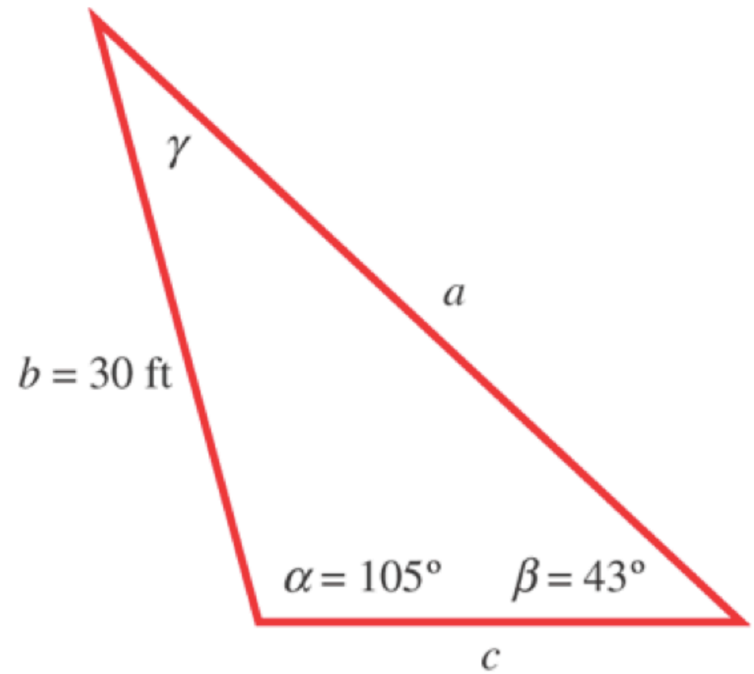
$b \approx 4.5\ m$

$c \approx 4.1\ m$

# 7.1 - Law of Sines



Practice: Solve the triangle (AAS)




$\gamma = 32°$

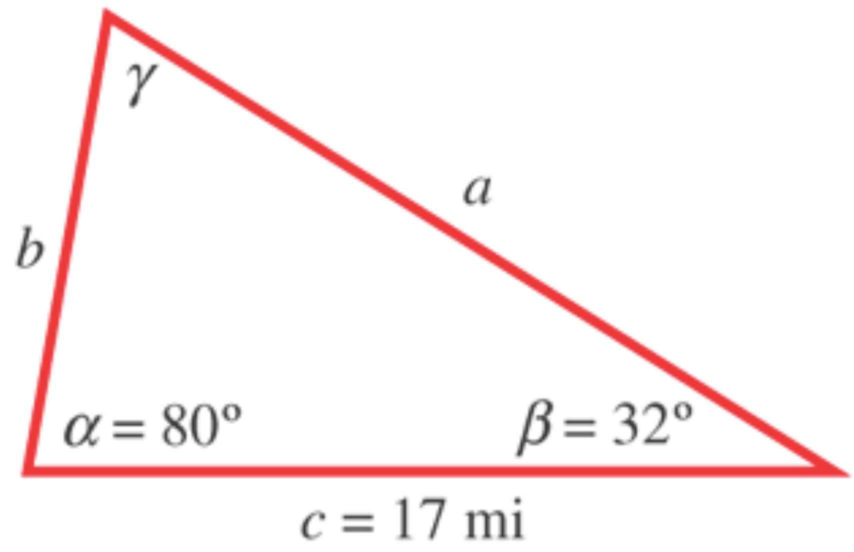
$a \approx 42.5\ ft$

$c \approx 23\ ft$

# 7.1 - Law of Sines



Solve the triangle (ASA)




$\gamma = 68°$

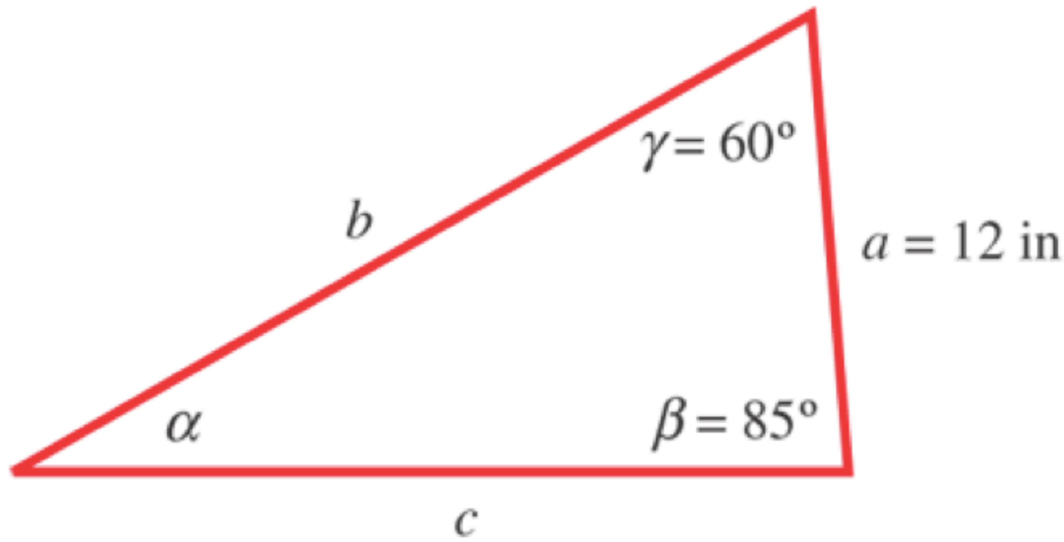
$b \approx 9.7\ mi$

$a \approx 18\ mi$

# 7.1 - Law of Sines



Practice: Solve the triangle (ASA)




$\alpha = 35°$

$b \approx 21\ in$

$c \approx 18\ in$

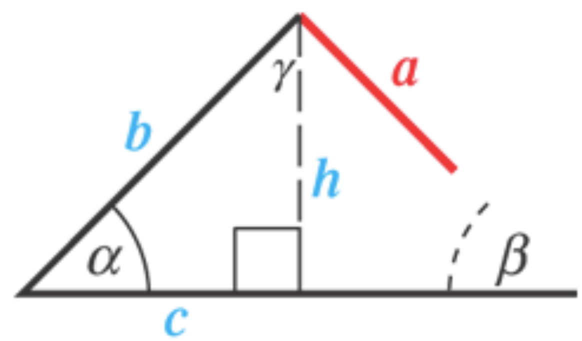
# 7.1 - Law of Sines



SSA will have one of three solutions given a, b, and $\alpha$.

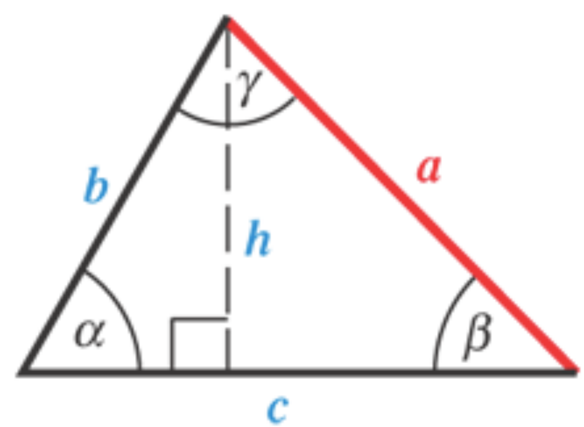
1. **No solution** when the height h is greater than the opposite side a.

$$h = b \sin \alpha \qquad h > a$$




2. **One solution** when side a is greater than or equal to side b.

$$a \geq b$$



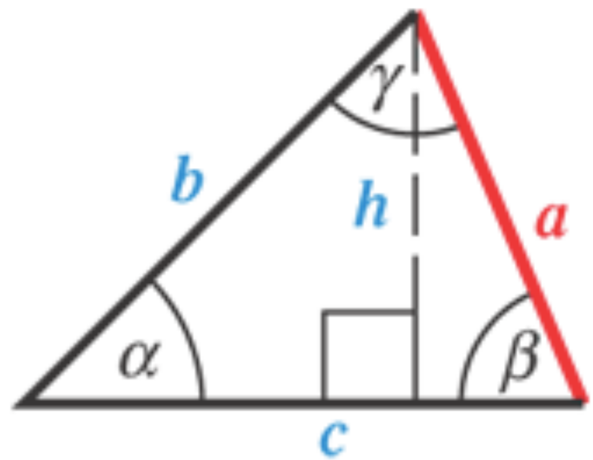
# 7.1 - Law of Sines

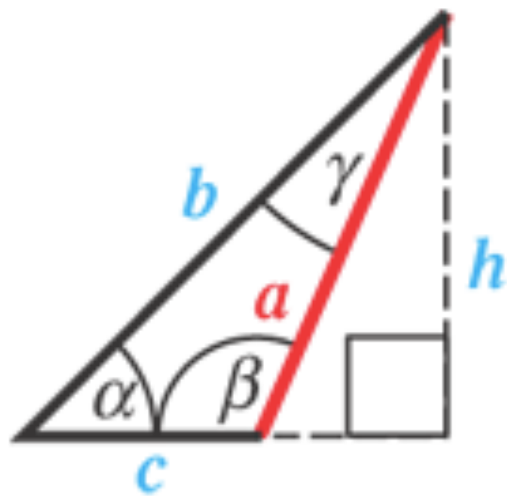


SSA will have one of several solutions. Given a, b, and $\alpha$.

3. **Two solutions** when a is greater than the height h and less than b. (Ambiguous case)

$$h < a < b$$



# 7.1 - Law of Sines



(SSA) Solve the triangle $a = 23$ feet, $b = 11$ feet, and $\alpha = 122^\circ$.

$a > b$ One solution

$\beta \approx 24^\circ$

$\gamma \approx 34^\circ$

$c \approx 15\ ft$

Practice: Solve the triangle $\alpha = 133^\circ$, $a = 48$ millimeters, and $c = 17$ millimeters.

$\beta \approx 32^\circ$

$\gamma \approx 15^\circ$

$b \approx 35\ mm$

# 7.1 - Law of Sines



(SSA) Solve the triangle $a = 8.1$ meters, $b = 8.3$ meters, and $\alpha = 72^\circ$.

$$h = 8.3 \sin 72^\circ \approx 7.89$$

$h < a < b$ Two solutions

$\beta_1 \approx 77^\circ$ $\beta_2 \approx 103^\circ$

$\gamma_1 \approx 31^\circ$ $\gamma_2 \approx 5^\circ$

$c_1 \approx 4.4\ m$ $c_2 \approx 0.74\ m$

# 7.1 - Law of Sines



(SSA) Solve the triangle $a = 6$, $b = 8$, and $\alpha = 107°$.

$$h = 8 \sin 107° \approx 7.65$$

$$a < h$$

**No triangle!!!**

# 7.1 - Law of Sines



Tower of Pisa: How far does it lean?


$\approx 5°$

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